Credit Concentration Risk :
Extended Multi-Factor Adjustment IRB Model

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Contents

Abstract

Credit concentration risk models were studied from 2000. Those studies were measured obligor (debtor) concentration (Name Concentration), Sector Concentration, Contagion Concentration models separately. Through integration of those models, we suggest consolidated one Extended Multi-Factor Adjstment model to calculate Name Concentration, Sector Concentration, and Contagion Concentration. This model has Closed Form Solution and recognizes fairly well in Name Concentration, Sector Concentration and Contagion Concentration.

Keywords : Credit Concentration Risk, Name Concentration, Sector Concentration, Contagion, Granularity Adjustment, Multi-Factor Adjustment, Extended Multi-Factor Adjustment

1. Introduction

Among credit risk measurements from Basel II, IRB model is based on these four assumptions. First, portfolio is consistested by small exposures, therefore, it assumes that individual asset's non-systematic risk is completely decentralized. Second, it assumes, all individual asset return is explained by one systematic factor. Generally, this kind of model is called one-factor model. Third, distribution of individual asset return is following normal distribution. Forth, LGD is constant. Among these four assumptions, first and second assumptions are mostly related to credit concentration risk. Now, credit concentration risk measurement models are developing as mitigating these assumptions. IRB model assumes non-systematic factors are completely decentralized in asset return. Therefore, it has the same VaR ratio for assets under the same condition but have different exposures. In addition, due to one-factor model, it can not explain assets which are influenced by different risk factors. Finally, models which are mainly used in financial institutes do not recongnized contagion effect. This is not the only limitation of IRB model but it is related to credit concentration risk.

According to BCBS (2006c), IRB model related to credit concentration risk issues are classified as following three;

1) Name Concentration

This is the concentration when individual obligor's exposure is big. As we assume that portfolio's individual exposure is unlimitedly small exposure (assume that number of obligors are unlimited). Due to this assumption as we assume that risk from non-systematic factors are completely diversified which means that cannot be calculated risk. Both one obligor has huge amount of exposure and the same amount of exposure is taken by so many obligors are treated equally in IRB model. Therefore, when individual obligor's exposure is huge, IRB model underestimates the risk. Here, we call this kind of credit concentration exposure as 'Exposure Concentration.'

2) Sector Concentration

Sector concentration risk is a concentration risk which occurs when specific obligor group is affected by the same risk factor. For example, if machinery industry is in its down turn session, their loss possibility by default is increasing. However, in IRB model, common risk factor, which affects to obligor is one factor. Therefore, this kind of risk can not be measured. Here, we call it as a sector concentration risk.

3) Contagion

Generally, this means default dependency. It is followed by common factor which are business dependency or legal dependency. This cannot be considered by IRB model. Here we call credit concentration risk due to contagion as contaigion concentration.

Through these three classifications, credit concentration risk measurement and model were studied. Based on three classification standard, credit concentration risk measurement model is divided into granularity adjustment, multi-factor adjustment, and contagion model. Granularity adjustment measures exposure concentration, multi-factor adjustment measures sector concentration and contagion measures contaigion.

1) Granularity Adjustment

Obligor concentration measurement models belong here. These are models that through mitigating non-systematic factor's complete diversification assumption approach in IRB model to measure credit concentration risk. Researches related to granularity adjustment are Gordy (2003), Gordy & Lutkebohmenrt (2006), Martin & Wilde (2002), and Emmer & Tasche (2005). Granularity Adjustment was firstly presented by Gordy (2003). Martin & Wilde (2002) presented closed-form to measure credit concentration.

Emmer & Tasche (2005) developed formula to compute individual assets' VaR contribution.

2) Multi-Factor Adjustment

Models for sector concentration risk belong here. These models expand one-factor assumption of Basel II into multi-factor to calculate credit concentration risk. Pykhtin (2004), Duellmann (2006), Garcia Cespedes (2005) are the related studies. Pykhtin (2004) created a model based on Creditmetrics's DM methodology and induced it with Martin & Wilde (2002) approach. Duellmann (2006) used expanded methodology of Davis & Lo (2001)'s BET model.

3) Contagion

Egloff, D. , Leippold, M., and Vanini. P. (2004), Fiori, and Foglia & Lannotti (2006) are reflecting contagion methodology. These are credit risk measurement models, which consider contagion effect. So, precisely, they are not the models that measure credit concentration risk due to contagion. From Egloff (2004) study, micro-structural dependency affects on tails of loss distribution, especially in non-dispersed portfolios. Therefore, micro-structural dependency has significant meaning in credit concentration risk.

Former models only measure certain credit concentration risks (exposure, sector, contagion concentration) as they mitigate IRB model's assumption. In other words, these were studied for measuring each of exposure, sector, and contagion concentration. Therefore, these models have weak point that they can not measure obligor, sector, and contagion as a whole. The objective of this study is measuring credit concentration risk using all of obligor, sector and contagion and developing extended model, which can calculate individual asset's exposure, sector, and contagion concentration. We call this model as EMFA (Extended Multi-Factor Adjustment Model, we will call it as EMFA)[2].

EMFA model is based on Pykhtin (2004)'s Multi-Factor Adjustment. Therefore, expand Pykhtin (2004)'s model to measure exposure, sector, contagion concentration. EMFA's characteristic is following;

1) EMFA (Extended Multi-Factor Adjustment) model is based on Pykhtin (2004)'s model, however, it is different from Pykhtin (2004)'s Granularity Adjustment. On the contrary, EMFA is similar to Emmer & Tasche (2005)'s granularity adjustment, which uses one-factor model to calculate exposure concentration. In individual asset part, Pykhtin (2004) included some lists, which should be included under sector concentration calculation[3], into Granularity Adjustment to underestimate exposure concentration risk amount.
2) Except conatigion model, former credit concentration risk measurement models are not reflecting contagion, however, EMFA reflects contagion effect. The way that EMFA reflects contagion effect is very similar to Egloff, D. , Leippold, M., and Vanini. P. (2004)'s but has simpler format than those. Egloff, D., Leippold, M., and Vanini. P. (2004) modeled business correlation among many obligors, however, EMFA model only reflects contagion effect on individual obligors only. This kind of approach is a special case of Egloff, D. , Leippold, M., and Vanini. P. (2004) model but in practice this is very easy. The reason is that, measuring how much do all the obligors, who consist portfolio, affect among others is practically diffeicult. However, recognizing groups that affect among others is relatively easy. In EMFA model, there is a good point that can explain default

[2] Contagion factor and Multi-Factor are independent. Contagion factor might only reflect one factor so, we call it Extended Multi-Factor Adjustment rather than generalized model(Contagion can affect both obligors of both parties, but our model only reflects contagion effect on individual industry).
[3] Pykhtin(2004) called it as a 'Systematic Term.'

contagion effect consequently, due to random factors among various sectors, which classified by multi-factor.

3) EMFA model can measure exposure, sector, and contagion concentration risk of individual asset. Preexisting credit concentration risk studies are mainly focused on measuring credit concentration risk for whole portfolio. Pykhtin(2004) also suggests credit concentration risk measurement model for whole portfolio, however, he did not suggest a method for individual asset. Using Marginal VaR concept, EMFA model can measure individual asset's credit concentration risk. This methodology is the same as Emmer & Tasche (2005)'s model, which uses one-factor model to calculate exposure concentration. Measuring individual asset's credi concentration risk makes it easy to manage credit concentration risk.

In this study, I had an actual analysis on whether EMFA can recognize credit concentration risk practically or not. In general, comparing result from Monte Carlo Simulation and result from EMFA, however, I set up a bench mark portfolio, which does not have credit concentration risk that I can compare assumable portfolios each has exposure concentration, sector concentration, or contagion concentration. The result shows that EMFA recognizes credit concentration risk very well. Of course, this result cannot ensure preciseness but managing credit concentration risk, this means that EMFA model can be very useful.

This treatise consists of five parts. In part 1, research precedence studies, which are bases of EMFA model. In part 2, suggest credit concentration risk measurement model based on obligor, sector, and contagion concentration. In part 3, explain how to estimate parameters for model. In part 4, describe the result of an actual analysis using measurement model. In part 5, this is the last part that concludes this treatise.

2. Precedence study

2.1. Emmer & Tasche(2005)'s Granularity Adjustment

Emmer & Tasche (2005) suggested exposure concentration measurement model using one-factor. Emmer & Tasche (2005) induced their model using the same approach as Pykhtin (2004), Martin & Wilde(2002), Gourieroux, C., J P Laurent and O Scaillet (2000). They also suggested exposure concentration calculation model by individual asset unit using Marginal VaR idea. Following is the model, which shows how Emmer & Tasche (2005) calculate portfolio exposure concentration risk.

$l_\alpha(L)$ is the actual VaR Ratio using one-factor and $l_\alpha(\overline{L})$ means VaR Ratio under assumption of Infinitely Fine-Grained. Follwing expression assumes that LGD=1.

$$l_\alpha(L) - l_\alpha(\overline{L}) \approx \left(\sum_{i=1}^{n} \left[w_i^2 \sqrt{\frac{R_i}{1-R_i}} n\left(\frac{N^{-1}(PD_i) - \sqrt{R_i} N^{-1}(1-0.999)}{\sqrt{1-R_i}} \right) \left(1 - 2N\left(\frac{N^{-1}(PD_i) - \sqrt{R_i} N^{-1}(1-0.999)}{\sqrt{1-R_i}} \right) \right) \right] \right.$$

$$\left. + \left(N^{-1}(1-0.999) + \frac{l''_\alpha(\overline{L})}{l'_\alpha(\overline{L})} \right) \sum_{i=1}^{n} w_i^2 \left(N\left(\frac{N^{-1}(PD_i) - \sqrt{R_i} N^{-1}(1-0.999)}{\sqrt{1-R_i}} \right) - N\left(\frac{N^{-1}(PD_i) - \sqrt{R_i} N^{-1}(1-0.999)}{\sqrt{1-R_i}} \right)^2 \right) \right) \times \left(2 \times l'_\alpha(\overline{L}) \right)^{-1}$$

R_i means i asset's asset correlation in Basel.

2.2. Pykhtin (2004)'s Multi-Factor Adjustment

According to Pykhtin (2004), credit concentration risk for whole portfolio will be expressed as following;

$$l_\beta(L) - l_\beta(\overline{L}) = -\frac{1}{2l'(y)} \left[v'(y) - v(y) \left(\frac{l''(y)}{l'(y)} + y \right) \right] \Bigg|_{y = N^{-1}(1-\beta)} \quad (2.2.1)$$

IRB model is expressed as following;

$$l_\beta(y)=\sum_{i=1}^{N} e_i \cdot LGD_i \cdot p_i(y) \tag{2.2.2}$$

$$p_i(y)=N\left(\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}}\right)\Bigg| y=N^{-1}(1-\beta)$$

IRB model's the first and second differential coefficients are expressed as following;

$$l'_\beta(y)=\sum_{i=1}^{N} e_i \cdot LGD_i \cdot p'_i(y) \tag{2.2.3}$$

$$p'_i(y)=-\frac{a_i}{\sqrt{1-a_i^2}}\,n\left(\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}}\right)\Bigg| y=N^{-1}(1-\beta)$$

$$l''_\beta(y)=\sum_{i=1}^{N} e_i \cdot LGD_i \cdot p''_i(y) \tag{2.2.4}$$

$$p''_i(y)=-\frac{a_i}{\sqrt{1-a_i^2}}\,\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}}\,n\left(\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}}\right)\Bigg| y=N^{-1}(1-\beta)$$

At expression (2.2.1), $v(y)$ and $v'(y)$ are expressed as following;

$$v(y)=v_{GA}(y)+v_\infty(y) \tag{2.2.5}$$

$$v'(y)=v'_{GA}(y)+v'_\infty(y) \tag{2.2.6}$$

$$v_\infty(y)=\sum_{i=1}^{N}\sum_{j=1}^{N} e_i \cdot e_j \cdot LGD_i \cdot LGD_j \cdot \left[N_2\left(N^{-1}[p_i(y)],\, N^{-1}[p_j(y)],\, \rho_{ij}^{Y}\right)-p_i(y)p_j(y)\right]$$

$$v'_\infty(y)=2\sum_{i=1}^{N}\sum_{j=1}^{N} e_i \cdot e_j \cdot LGD_i \cdot LGD_j \cdot p'_i(y)\cdot\left[N\left(\frac{N^{-1}[p_j(y)]-\rho_{ij}^{Y}\ N^{-1}[p_i(y)]}{\sqrt{1-(\rho_{ij}^{Y})^2}}\right)-p_j(y)\right]$$

$$v_{GA}(y)=\sum_{i=1}^{N} e_i^2\left(LGD_i^2\cdot\left[p_i(y)-N_2\left(N^{-1}[p_i(y)],\, N^{-1}[p_i(y)],\, \rho_{ii}^{Y}\right)\right]+\sigma_i^2\ p_i(y)\right)$$

$$v'_{GA}(y)=\sum_{i=1}^{N} e_i^2 \cdot p'_i(y)\cdot\left(LGD_i^2\cdot\left[1-2\cdot N\left(\frac{N^{-1}[p_i(y)]-\rho_{ij}^{Y}\,N^{-1}[p_i(y)]}{\sqrt{1-(\rho_{ii}^{Y})^2}}\right)\right]+\sigma_i^2\right)$$

From Pykhtin (2004)'s study, $v_{GA}(y)$ and $v'_{GA}(y)$ mean exposure concentration(name concentration), $v_\infty(y)$ and $v'_\infty(y)$ are related to sector concentration.

Pykhtin(2004) suggests measurement method for expected shortfall, which is based on IRB model, and portfolio multi-factor adjustment expected shortfall as following;

$$ES_\beta(\bar{L}) = \frac{1}{1-\beta}\sum_{i=1}^{M} e_i \cdot LGD_i \cdot N_2\left[N^{-1}(PD_i), N^{-1}(1-\beta), a_i\right] \quad (2.2.7)$$

$$\Delta ES_\beta(L) = -\frac{1}{2(1-\beta)} n(y) \frac{v(y)}{l'(y)}\bigg|_{y = N^{-1}(1-\beta)} \quad (2.2.8)$$

2.3. Egloff, D., Leippold, M., and Vanini. P. (2004)

As Egloff and others (2004) pointed out, purely macrostructural models cannot reflect high default correlationship. Egloff and others (2004) developed a credit risk measurement model, which reflects macrostructual dependency and microstructural dependency among obligors in credit portfolio. This model also reflects microstructural dependency caused by common factors such as business dependency, legal dependency.

3. Extended Multi-Factor Adjustment IRB model

Let's assumen that individual assets in portfolio expressed as i, total number of assets expressed as N, explaining systematic factor expressed as k and total number of systematic factor expressed as M. For IRB model, infinitely-fine grained assumption was used. This assumes that portfolio, which is consisted of limited exposures,'s number of obligors are unlimited. This has meaning that unsystematic factors are diversified and only risk from systematic factors will be remaining. EMFA does not use this infinitely-fine grained assumption.

The same as Pykhtin (2004)'s MFA model, EMFA model can measure additional risk that IRB model can not do. Therefore, it only needs basic inputs (PD, LGD, EAD, Maturity and etc) as IRB model. Additionally, systematic factor and contagion related informations will be needed.

EMFA model inducing process is basically the same as Pykhtin (2004)'s. However, it reflects some of additional informations, it revises and extends Pykhtin (2004)'s MFA model. Before explaining EMFA model, I would like to define some of ideas here. $N(x)$ is standard normal distribution's cumulated density function, $n(x)$ is standard normal distribution's density function, $N^{-1}(p)$ means standard normal distribution's cumulated density function's reveresd function, and $N_2(\ )$ means bivariate normal distribution's cumulated density function. EMFA puts weight on exposure and it is expressed as $e_i = E_i \Big/ \sum_{j=1}^{N} E_j$ (N : total number of asset , E_i : i exposure of asset). While LGD is a constant in IRB model, it is random variable in EMFA and uses Pykhtin & Dev (2002)'s method on this matter. Pykhting (2004)'s model also selects the same LGD model to induce its model and EMFA follows that.

IRB model's asset return is following;

$$r_i = a_i \overline{Y} + \sqrt{1-a_i^2}\,\varsigma_i \tag{3.1}$$

r_i : i return of asset

$\overline{Y}$: i systematic factor of asset (in IRB model)

ς_i : i non-systematic factor of asset (asset characteristic in IRB model)

a_i : i correlation coefficient of asset (asset correlation in IRB model)

$\overline{Y}, \varsigma_i \sim N(0,1)$, $\overline{Y}$ and ς_i are independent

EMFA (Extended Multi-Factor Adjustment)'s asset return extends IRB model's asset return into multi-factor. It is the same with Pykhtin (2004) that extending systematic factor into multi-factor, however, it is different that classifies non-systematic factor into contagion factor and obligor's own factor in IRB model's asset earning model. Contagion factor creates correlation between mutually depending corporates by business dependency between individual obligors. For example, small and medium sized companies, which have business with Samsung, keep close relationships with Samsung's business showings. Therefore, when Samsung ended up as broke, the possibility of those related SMEs' default rate are also increasing. Here's the EMFA asset earning ratio model;

$$r_i = \omega_i Y_i + \sqrt{1-\omega_i^2}\,\varepsilon(X_i, \varepsilon_i) \qquad (3.2)$$

r_i : i return of asset

Y_i : i systematic factor

$\varepsilon(X_i, \varepsilon_i)$: i unsystematic factor

$Y_i, \varepsilon_i \sim N(0,1)$, Y_i and ε_i are independent

ω_i : systematic factor Loading

$N(\mu,\sigma)$ is normal distribution which have average of (μ,σ) and standard deviation of (μ,σ).

Systematic factor (Y_i) is expressed by multi-factor. Systematic factor (Y_i) is different in IRB model and EMFA model.

$$\text{EMFA: } Y_i = \alpha_{i1}Z_1 + \alpha_{i2}Z_2 + \cdots + \alpha_{iM}Z_M = \sum_{k=1}^{M} \alpha_{ik}Z_k\ ,\ \ Z_k \sim N(0,1) \tag{3.3}$$

$$\text{IRB: } Y_i = \overline{Y} \tag{3.4}$$

Z_i : multi-factor

As stated before, non-systematic factors in EMFA are 1) Contagion factor, 2) obligor's own factor. Contagion factor is expressed at expression (3.2)'s non-systematic factor $(\varepsilon(X_i, \varepsilon_i))$ as follows by contagion factor (X_i) and obligor's own factor (ε_i).

$$\varepsilon(X_i, \varepsilon_i) = g_i X_i + \sqrt{1 - g_i^2}\,\varepsilon_i \tag{3.5}$$

$$X_i = c_{i1}C_1 + c_{i2}C_2 + \cdots c_{iK}C_K = \sum_{k=1}^{K} c_{ij}C_k$$

g_i : contagion factor Loading

C_k : contagion factor , $corr(C_i, C_j) = 0$

$k = 1,2,\cdots,K$, K = total number of contagion factor

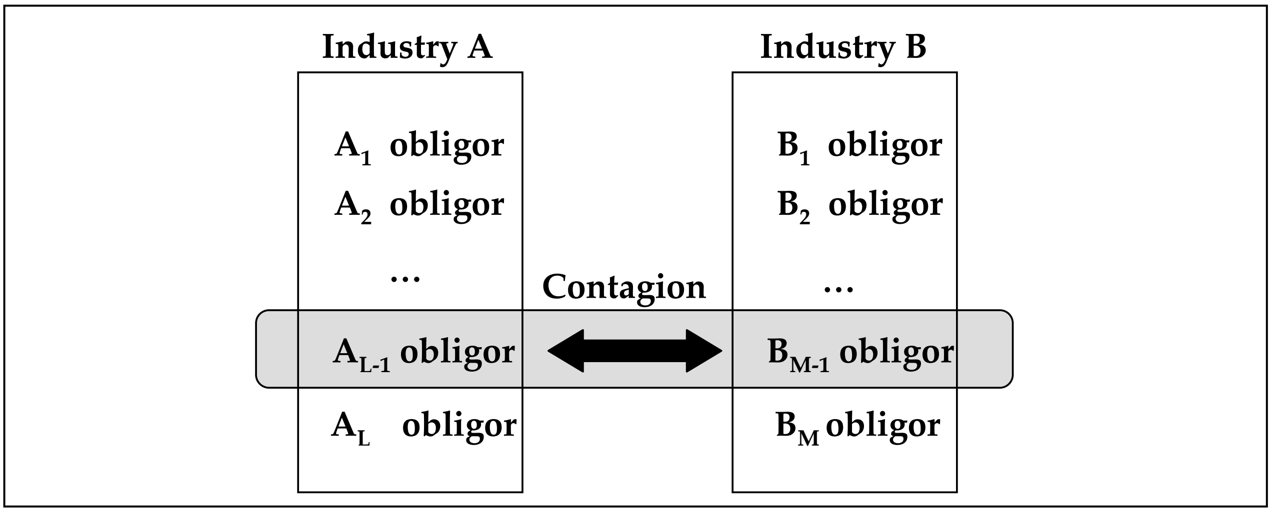
EMFA's asset earning ratio model reflects contagion factor in Pykhtin (2004)'s asset earning ratio model which only considers multi-factor. Therefore, Pykhtin (2004)'s conditional asset correlation (ρ_{ij}^{Y}) and EMFA's conditional asset correlation (ρ_{ij}^{Y+G}) are following

$$\rho_{ij}^{Y} = \frac{r_i r_j \sum_{k=1}^{M} \alpha_{ik}\alpha_{jk} - a_i a_j}{\sqrt{(1-a_i^2)(1-a_j^2)}} \tag{3.6}$$

$$\rho_{ij}^{Y+C} = \frac{\omega_i \omega_j \sum_{k=1}^{M} \alpha_{ik}\alpha_{jk} + \sqrt{1-\omega_i^2}\sqrt{1-\omega_j^2}\, g_i g_j \sum_{k=1}^{K} c_{ik}c_{jk} - a_i a_j}{\sqrt{(1-a_i^2)(1-a_j^2)}} \tag{3.7}$$

Finally, expression (3.6) means conditional asset correlation which considers multi-factor only in asset earning ratio and expression (3.7) means conditional asset correlation which considers both multi-factor and contagion for asset earning ratio.

Contagion finally can explain how the defect can be contaged among obligors other than multi-factor as follows. In other words, it makes the explanation possible that contagion between industry sectors with factors other than industry correlation. In the end, contagion reflects



relations caused by business dependency between obligors. For example, in Korea, obligors under one sector has contagion possibility as they mutually invest to among one another.

EMFA model which calculates credit concentration risk by individual asset is following. For detailed expression, see appendix A.

$$\frac{d(l_\beta(L))}{de_i} - \frac{d(l_\beta(\bar{L}))}{de_i} = \frac{d(l_\beta(L))}{de_i} - LGD_i \cdot p_i(y)$$
$$\approx -\frac{1}{2}\left[\frac{B\cdot l' - v'\cdot A}{(l')^2} - \frac{C\cdot l' - l''\cdot A}{(l')^2}\cdot\frac{v}{l'} - \left(\frac{l''}{l'}+y\right)\left(\frac{D\cdot l' - v\cdot A}{(l')^2}\right)\right] \qquad (3.8)$$

$$A = \frac{dl'(y)}{de_i} = LGD_i \cdot p_i'(y)$$

$$B = \frac{dv'(y)}{de_i} = \frac{dv'_{GA}(y)}{de_i} + \frac{dv'_{\infty}(y)}{de_i}$$

$$C = \frac{dl''(y)}{de_i} = LGD_i \cdot p_i''(y)$$

$$D=\frac{dv(y)}{de_i}=\frac{dv_{GA}(y)}{de_i}+\frac{dv_{\infty}(y)}{de_i}$$

$$\frac{dv_{GA}(y)}{de_i}=2\cdot e_i\cdot\left(LGD_i^2\cdot[p_i(y)-p_i(y)p_i(y)]+\sigma_i^2 p_i(y)\right)$$

$$\frac{dv'_{GA}(y)}{de_i}=2\cdot e_i\cdot p'_i(y)\cdot\left(LGD_i^2\cdot[1-2\cdot p_i(y)]+\sigma_i^2\right)$$

$$\frac{dv_{\infty}(y)}{de_i}=2\cdot LGD_i\cdot\sum_{j=1}^{N}e_j LGD_j\left\{N_2\left(N^{-1}[p_i(y)],N^{-1}[p_j(y)],\rho_{ij}^{Y+C}\right)-p_i(y)p_j(y)\right\}$$
$$-2\cdot e_i\cdot\sum_{i=1}^{N}LGD_i^2\left\{N_2\left(N^{-1}[p_i(y)],N^{-1}[p_i(y)],\rho_{ii}^{Y+C}\right)-p_i(y)^2\right\}$$

$$\frac{dv'_{\infty}(y)}{de_i}=2\cdot 2\cdot LGD_i\cdot\sum_{j=1}^{N}e_j\cdot LGD_j\cdot p'_i(y)\left\{N\left(\frac{N^{-1}[p_j(y)]-\rho_{ij}^{Y+C}N^{-1}[p_i(y)]}{\sqrt{1-\left(\rho_{ij}^{Y+G}\right)^2}}\right)-p_j(y)\right\}$$
$$-2\cdot e_i\cdot p'_i(y)\left(LGD_i^2\left[2N\left(\frac{N^{-1}[p_j(y)]-\rho_{ii}^{Y+C}N^{-1}[p_i(y)]}{\sqrt{1-\left(\rho_{ii}^{Y+C}\right)^2}}\right)-2p_i(y)\right]\right)$$

Make expression (3.8) simple,

$$\frac{d(l_\beta(L))}{de_i}-\frac{d(l_\beta(\bar{L}))}{de_i}=F\left(PD,LGD,e,\rho^{Y+C}\right)\qquad(3.9)$$

Contagion concentration can be calculated as follows. From muti-factor and contagion reflected concentration risk amount minus multi-factor only concentration risk amount.

$$Contagion=F\left(PD,LGD,e,\rho^{Y+C}\right)-F\left(PD,LGD,e,\rho^{Y}\right)\qquad(3.10)$$

ρ^{Y} : Conditional Asset Correlation reflecting multi-factor

ρ^{Y+C} : Conditional Asset Correlation reflecting multi-factor and contagion

Finally we divide concentration risk and diversification effect. From this we could separately recognize diversification effect which comes from extending systematic factor into multi-factor. Sector concentration risk can be calculated as follows. The result from sector concentration expression, when it is a positive number, it will be considered concentration risk and when it is a negative number, it will be considered as diversification effect. In addition, this will be calculated as individual asset unit, it is quiet easy to reflect Basel II's maturity effect.

EMFA model can apply the expected shortfall measuring method, which was suggested by Pykhtin (2004). Therefore it will be easy to calculate expected shortfall from EMFA.

4. Model Estimation

Define industry by systematic factor, input elements for EMFA are industry classification (classification of systematic factor) correlation among industries (correlation among systematic factors), systematic factor Loading, contagion factor Loading, LGD standard deviation, PD, LGD, EAD, and maturity. All these factors will be defined under model estimation.

4.1. Industry classification and correlation among industries

4.1.1. Industry classification

Generally, industry used for multi-factor. Here, cacluate correlation among industry using KOSPI industry index and for industry classification, use KOSPI industry category index classification. KOSPI industry category index is following;

Food&beverages	Nonmetalic minerals	Transportation equip.	Telecommunication	Service
Fabric&clothing	Steel. Metal	Distribution	Finance	Manufacturing
Wood&Papaer	Machinary	Electronics&gas	Banking	
Chemistry	Electricity.Electron	Construction	Securities	
Pharmaceutical	Medical equipment	Transportation storage	Insurance	

<Table 1> Industry classification

4.1.2. Correlation among industry

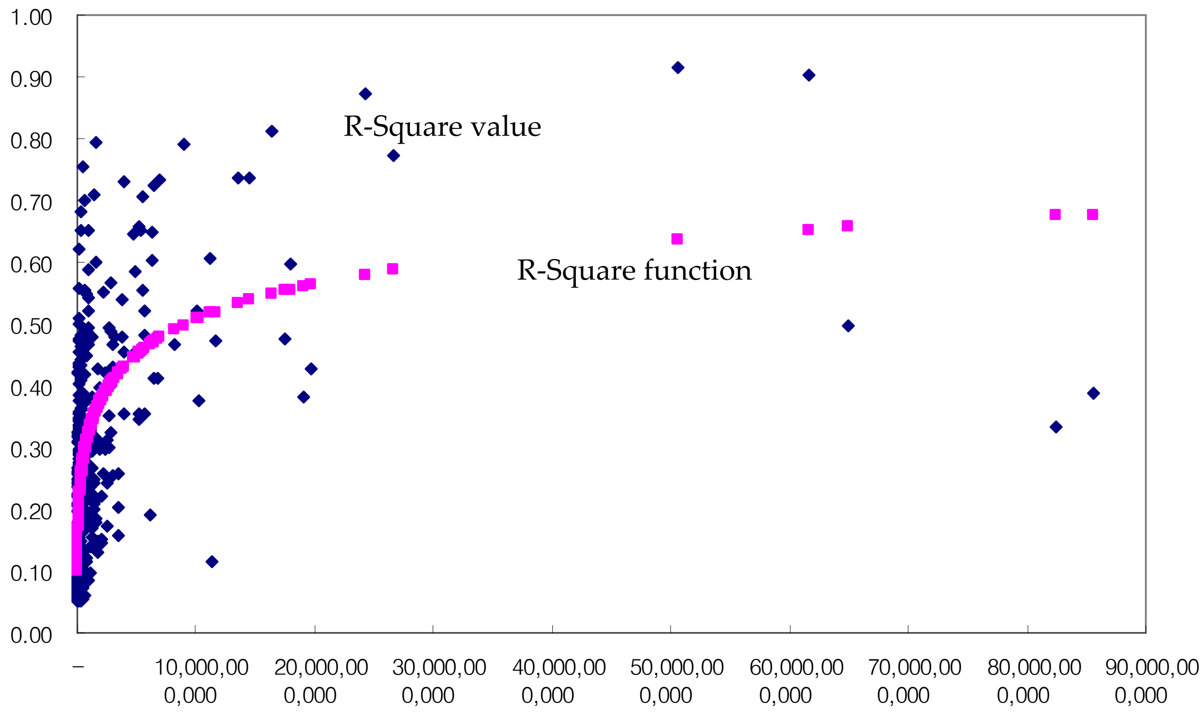
To calculate correlation among industries, use KOSPI industry category index. Terms of data to calculate correlation among industries is from Jan. 2000 through November 2006. Monthly data was used and for correlation coefficient among industries, see <appendix B>.

4.2. Factor Loading

4.2.1. Systematic Factor Loading

Analysis was hold for 556 corporates from KOSPI market and monthly closing rate of last date of month from Jan. 2000 to Dec. 2006 was used. Using closing rate and industry rate of observation term, calculate Beta by regression analysis.

Beta can be appeared in total asset's function and the way how to show relation ship between Beta and total asset is the same as credit metrics method. CreditMetrics uses R-Square and to calculate systematic factor Loading, I am following this method. Following shows the relation ship between total asset (based on Dec. 2006) and R-Square by its industry.



< Figure 1 > Total asset and R-Square(X is total asset by Won, Y is R-Square), R-Square function

R-Square function follows least square method to estimate and the result is following.

$$R^2 = \frac{1}{1 + A^{-0.3314} e^{7.60255}} \text{, A : Total of asset} \qquad (4.2.1.1)$$

4.2.2. Contagion Factor Loading

Estimating contagion factor Loading means the degree of how will obligors be affected by contagion factor. Contagion factor can establish many things theoretically but it is difficult to define contagion factor. In Korea, obligors under similar categories have correlated dependency on asset or business. Therefore, in analysis, I set up contagion factor by interrelated industries. Contagion factor Loading is obligors' total asset/ total assets of interrelated industries. This assumes that in interrelated industries, which has higher total assets affect more than lower total assets companies. This also appears in stock price, in electricity category, Samsung has biggest share and influence. Therefore, asset's volume is an important factor to define factor Loading.

4.3. Mediation of asset correlation

Whole portfolio's asset correlation of every model, which is based on multi-factor, will be changed due to number of industry classification or systematic factor Loading. In other words, increasing number of industry or decreasing systematic factor Loading, there is probability to measure credit concentration risk too small by individual financial institutes. EMFA has the same problem. When input element is decided, under Basel II, asset correlation will be finalized for whole portfolio. However, as it extends into multi-factor, there is possibility asset correlation can be shrinked for whole portfolio. However, it does not suggest standards towards number of industries or systematic factor Loading in case applying multi-factor models in regulated capital aspect. Also appropriateness study on this has not been done, yet.

To solve this kind of issues, adjust whole portfolio's correlation with whole portfolio's correlation using Basel II asset correlation and EMFA's whole portfolio asset correlation. This keeps correlation structure to recognize sector concentration while maintaining whole portfolio's correlation as the same as Basel II to keep whole portfolio's total asset correlation coefficient.

As we assume that individual asset's earning ratio distribution follows normal distribution in EMFA and IRB, therefore, portfolio asset correlation is following;

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} e_i \sigma_i \rho_{ij} \sigma_j e_i = \sum_{i=1}^{N}\sum_{j=1}^{N} e_i \rho_{ij} e_i = \rho_p \qquad (4.3.1)$$

σ_p^2 : portfolio variance

e_i : i exposure weight of asset

σ_i : i standard deviation of asset

ρ_{ij} : i asset and j asset's correlation

Calculation of correlation adjustment coefficient is following;

$$\rho_c = \rho_{IRB} - \rho_{EMFA} \qquad (4.3.2)$$

ρ_c : constant correlation adjustment coefficient

ρ_{IRB} : asset correlation by IRB model

ρ_{EMFA} : asset correlation by EMFA model

To calculate correlation adjustment coefficient, we are not using conditional asset correlation expression which considers multi-factor and contagion factor at expression (3.7), use expression (3.6) and calculate ρ_{EMFA} to get correlation adjustment coefficient.

Using expression (3.7), which is EMFA model's conditional asset correlation, and correlation adjustment coefficient, make an adjustment as following, portfolio asset correlation by IRB model and EMFA model become the same.

$$\rho_{ij}^{Y+C} = \frac{\omega_i \omega_j \sum_{k=1}^{M} \alpha_{ik} \alpha_{jk} + \rho_c + \sqrt{1-\omega_i^2}\sqrt{1-\omega_j^2}\, g_i g_j \sum_{t=1}^{T} c_{it} c_{jt} - a_i a_j}{\sqrt{\left(1-a_i^2\right)\left(1-a_j^2\right)}} \qquad (4.3.3)$$

4.4. LGD standard deviation

Pykhtin & Dev(2002)'s method will be applied for volatility of LGD.

$$\text{LGD standard deviation} = \sigma_{LGD} = \frac{1}{2}\sqrt{LGD(1-LGD)} \qquad (4.3.1)$$

5. Analysis

5.1. Analysis Case A

5.1.1. Data

All of the obligors must have the same exposure, correlation of industry should be one and there should be no concentration risk in portfolio, which has an asset correlation in IRB model. From our analysis, first, based on these bench mark portfolio, compare if assumed portfolio with huge exposure obligors have exposure concentration. Second, change only industrial correlation structure to see if there is any sector concentration risk among high correlation industries.

Following is the set up for assumed portfolio. Common input elements for assumed portfolio analysis are number of obligor (50,000), PD=1%, LGD=4 and Maturity=1 year0%

Classification	Name of portfolio	Set up details	References
Bench mark	Bench mark portfolio	☐All obligors are the same ☐Coefficient of correlation among industry is “1” ☐Systematic factor Loading is the same as IRB asset correlation ☐No contagion factor	Standard portfolio
Exposure concentration	Portfolio 1	☐10% of total obligors have 30% of total exposure (exposure for this 10% is the same and rest of the obligors have the same exposure) ☐Coefficient of correlation among industry is “1” ☐Systematic factor Loading is the same as IRB asset correlation ☐No contagion factor	Concentration risk changes along with EAD disribution

Classification	Name of portfolio	Set up details	References
	Portfolio 2	□8% of total obligors have 30% of total exposure (exposure for this 8% is the same and rest of the obligors have the same exposure) □Coefficient of correlation among industry is "1" □Systematic factor Loading is the same as IRB asset correlation □No contagion factor	
	Portfolio 3	□5% of total obligors have 30% of total exposure (exposure for this 5% is the same and rest of the obligors have the same exposure) □Coefficient of correlation among industry is "1" □Systematic factor Loading is the same as IRB asset correlation □No contagion factor	
	Portfolio 4	□1% of total obligors have 30% of total exposure (exposure for this 1% is the same and rest of the obligors have the same exposure) □Coefficient of correlation among industry is "1" □Systematic factor Loading is the same as IRB asset correlation □No contagion factor	
Sector concentration	Portfolio 5	□All of the obligors have the same exposure □Coefficient of correlation among industry is "1" □Systematic factor Loading is R-Square function from 4.2.1 □No contagion factor	Followed by coefficient correation increasing among industry, the amount of concentration

Classification	Name of portfolio	Set up details	References
	Portfolio 6	☐All of the obligors have the same exposure ☐Coefficient correlation = 0.5 ☐Systematic factor Loading is R-Square function from 4.2.1 ☐No contagion factor	risk increasing ➔ ES increasing rate is faster than VaR ➔ compare with contagion
	Portfolio 7	☐All of the obligors have the same exposure ☐Coefficient correlation = 0.6 ☐Systematic factor Loading is R-Square function from 4.2.1 ☐No contagion factor	
	Portfolio 8	☐All of the obligors have the same exposure ☐Coefficient correlation = 0.7 ☐Systematic factor Loading is R-Square function from 4.2.1 ☐No contagion factor	
	Portfolio 9	☐All of the obligors have the same exposure ☐Coefficient correlation = 0.8 ☐Systematic factor Loading is R-Square function from 4.2.1 ☐No contagion factor	
	Portfolio 10	☐All of the obligors have the same exposure ☐Coefficient correlation = 0.9 ☐Systematic factor Loading is R-Square function from 4.2.1 ☐No contagion factor	

Classification	Name of portfolio	Set up details	References
	Portfolio 11	☐All of the obligors have the same exposure ☐Total of 22 industry, half of them (group A) have "0.2" coefficient correlation and rest of them (group B) have "0.8."Correlation between these two groups are "0.5" ☐ Systematic factor Loading is R-Square function from 4.2.1 ☐No contagion factor	Effect of contagion factor Loading on concentration risk ➔ ES increasing rate is faster than VaR
Contagion concentration	Portfolio 12	☐All of the obligors have the same exposure ☐Total of 22 industry, half of them (group A) have "0.2" coefficient correlation and rest of them (group B) have "0.8."Correlation between these two groups are "0.5" ☐Systematic factor Loading is R-Square function from 4.2.1 ☐One factor contagion factor is used and the loadage is 2%	
	Portfolio 13	☐All of the obligors have the same exposure ☐Total of 22 industry, half of them (group A) have "0.2" coefficient correlation and rest of them (group B) have "0.8."Correlation between these two groups are "0.5" ☐Systematic factor Loading is R-Square function from 4.2.1 ☐One factor contagion factor is used and the loadage is 4%	

Classification	Name of portfolio	Set up details	References
	Portfolio 14	☐All of the obligors have the same exposure ☐Total of 22 industry, half of them (group A) have “0.2” coefficient correlation and rest of them (group B) have “0.8.” Correlation between these two groups are “0.5” ☐Systematic factor Loading is R-Square function from 4.2.1 ☐One factor contagion factor is used and the loadage is 6%	
	Portfolio 15	☐All of the obligors have the same exposure ☐Total of 22 industry, half of them (group A) have “0.2” coefficient correlation and rest of them (group B) have “0.8.” Correlation between these two groups are “0.5” ☐Systematic factor Loading is R-Square function from 4.2.1 ☐One factor contagion factor is used and the loadage is 10%	

<Table 2> Assumed portfolio summary

From <Table 2>, gave correlation structure dividing portfolio 6&7 into two groups. Following defines[4]. group A and group B.

Group A		Group B	
Food & beverages	Nonmetalic minerals	Distribution	Finance
Fabric & clothes	Steel. Metal	Electricity & Gas	Banking
Wood & paper	Machinery	Construction	Securities
Chemistry	Electricity. Electron	Transportation & storage	Insurance

[4] Groups were divided randomly.

Pharmaceutical	Medical equipment	Telecommunication	Service industry
	Transportation equipment		Manufacturing

<Table 3> Industrial group classification

5.1.2. Analysis result

Under the same condition, when we increase specific obligors' exposure weight, concentration risk was also increased. Distribute the same amount of exposures which is 30% of total exposure to specific obligors. Therefore, this means that even the total portfolio amount is not changed, according to portfolio's EAD distribution, exposure concentration appears differently. Following <Table 4> and <Table 5> show VaR ratio and shortfall ratio of exposure, sector, contagion and total credit concentration by each portfolio.

Classification	Herfindahl index	Exposure (VaR Ratio)	Sector (VaR Ratio)	Contagion (VaR Ratio)	Total credit (VaR Ratio)
Bench mark	0.00002	0.001455%	0.789814%	0.000000%	0.791269%
Portfolio 1	0.0000289	0.002104%	0.790441%	0.000000%	0.792545%
Portfolio 2	0.00003315	0.002553%	0.790785%	0.000000%	0.793338%
Portfolio 3	0.000046316	0.004236%	0.791667%	0.000000%	0.795902%
Portfolio 4	0.0001899	0.022188%	0.797636%	0.000000%	0.819824%
Portfolio 5	0.00002	0.001455%	0.008186%	0.000000%	0.009641%
Portfolio 6	0.00002	0.001455%	0.000000%	0.000000%	0.001455%
Portfolio 7	0.00002	0.001455%	0.000053%	0.000000%	0.001509%
Portfolio 8	0.00002	0.001455%	0.000422%	0.000000%	0.001878%
Portfolio 9	0.00002	0.001455%	0.001370%	0.000000%	0.002825%
Portfolio 10	0.00002	0.001455%	0.002676%	0.000000%	0.004131%
Portfolio 11	0.00002	0.001455%	0.000166%	0.000000%	0.001622%
Portfolio 12	0.00002	0.001455%	0.000166%	0.387039%	0.388660%
Portfolio 13	0.00002	0.001455%	0.000166%	0.794738%	0.796360%
Portfolio 14	0.00002	0.001455%	0.000874%	1.223213%	1.225542%
Portfolio 15	0.00002	0.001455%	0.007120%	2.142947%	2.151522%

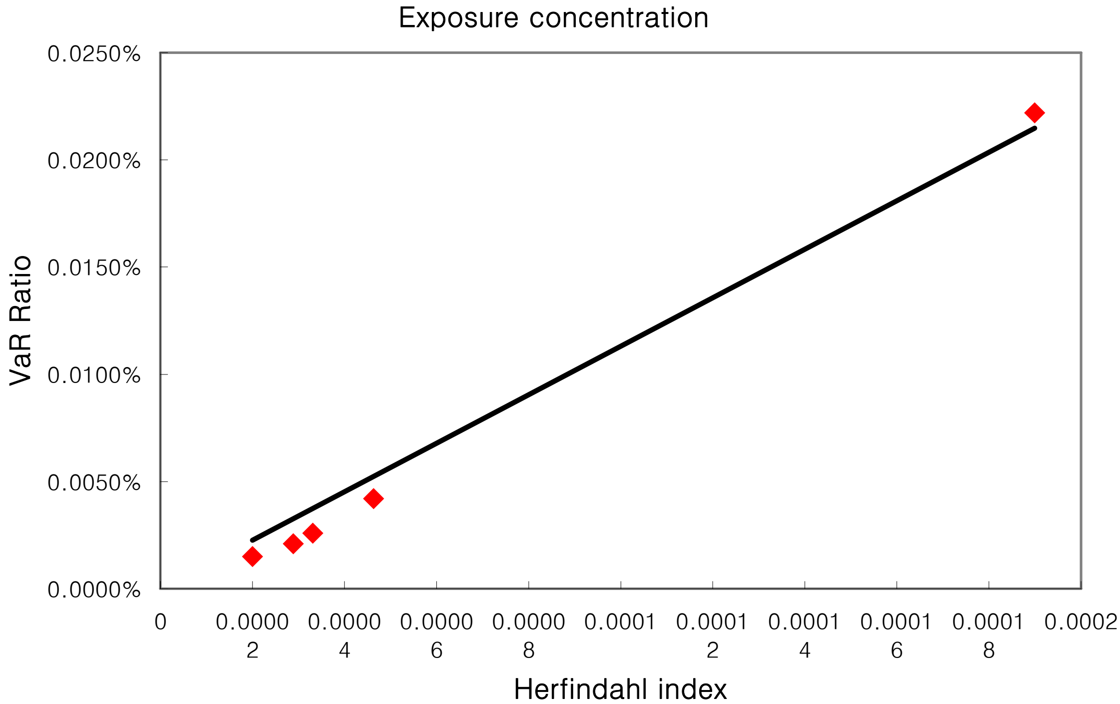
< Table 4 > Portfolio 1~15's VaR ratio of exposure, sector, contagion and total credit concentration

Classification	Herfindahl index	Exposure (ES Ratio)	Sector (ES Ratio)	Contagion (ES Ratio)	Total credit (ES Ratio)
Bench mark	0.00002	0.001647%	1.026636%	0.000000%	1.028283%
Portfolio 1	0.0000289	0.002381%	1.027460%	0.000000%	1.029841%
Portfolio 2	0.00003315	0.002890%	1.027911%	0.000000%	1.030801%
Portfolio 3	0.000046316	0.004794%	1.029069%	0.000000%	1.033864%
Portfolio 4	0.0001899	0.025126%	1.036906%	0.000000%	1.062033%
Portfolio 5	0.00002	0.001647%	0.010888%	0.000000%	0.012535%
Portfolio 6	0.00002	0.001647%	0.000000%	0.000000%	0.001647%
Portfolio 7	0.00002	0.001647%	0.000078%	0.000000%	0.001725%
Portfolio 8	0.00002	0.001647%	0.000592%	0.000000%	0.002239%
Portfolio 9	0.00002	0.001647%	0.001882%	0.000000%	0.003529%
Portfolio 10	0.00002	0.001647%	0.003628%	0.000000%	0.005275%
Portfolio 11	0.00002	0.001647%	0.000232%	0.000000%	0.001879%
Portfolio 12	0.00002	0.001647%	0.000232%	0.540364%	0.542244%
Portfolio 13	0.00002	0.001647%	0.000232%	1.102580%	1.104459%
Portfolio 14	0.00002	0.001647%	0.000232%	1.686617%	1.688496%
Portfolio 15	0.00002	0.001647%	0.000232%	2.920114%	2.921993%

< Table 5 > Portfolio 1~15's expected shortfall ratio of exposure, sector, contagion and total credit concentration.

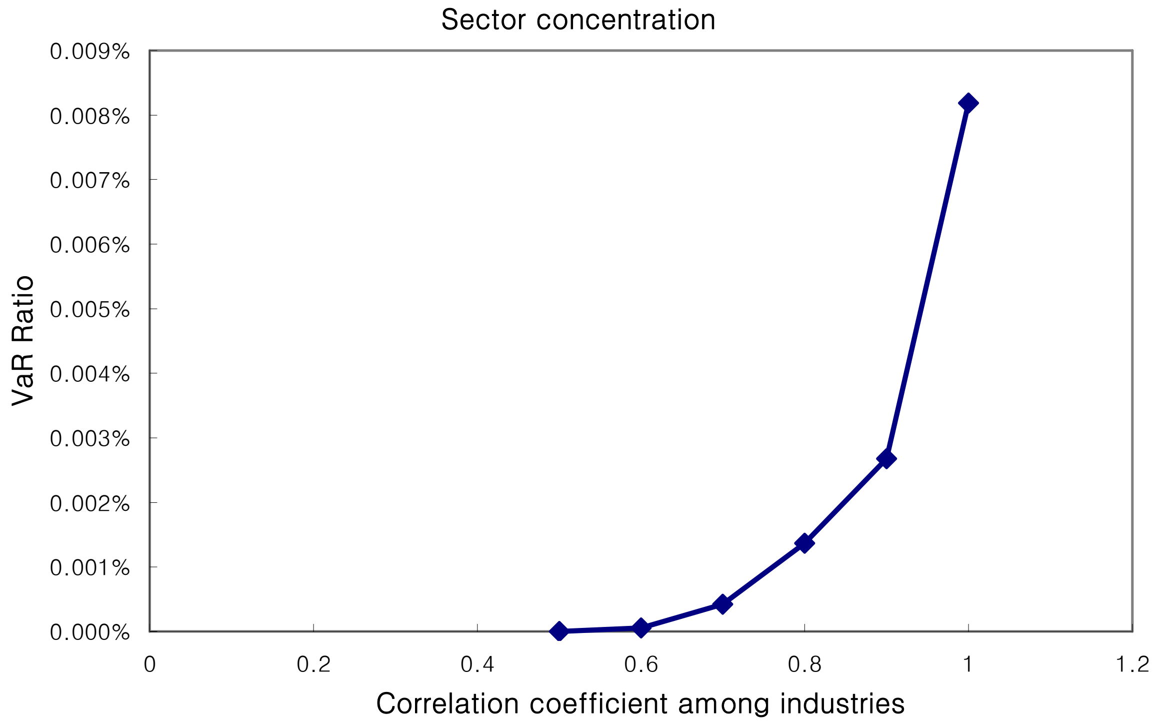
As you can see from <Table 4>and <Table 5>, concentration risk by expected shortfall is bigger than by VaR.

<Figure2> shows relationship between porfolio 1~4's exposure concentration and Herfindahl index. Portfolio 1~4 are homogeneous portfolios, which all the obligors have the same PD, LGD and maturity but different EAD. These homogeneous portfolioes show linear relationship between Herfindahl index and esposure concentration. The result is matched with Gordy (2003)'s study result.



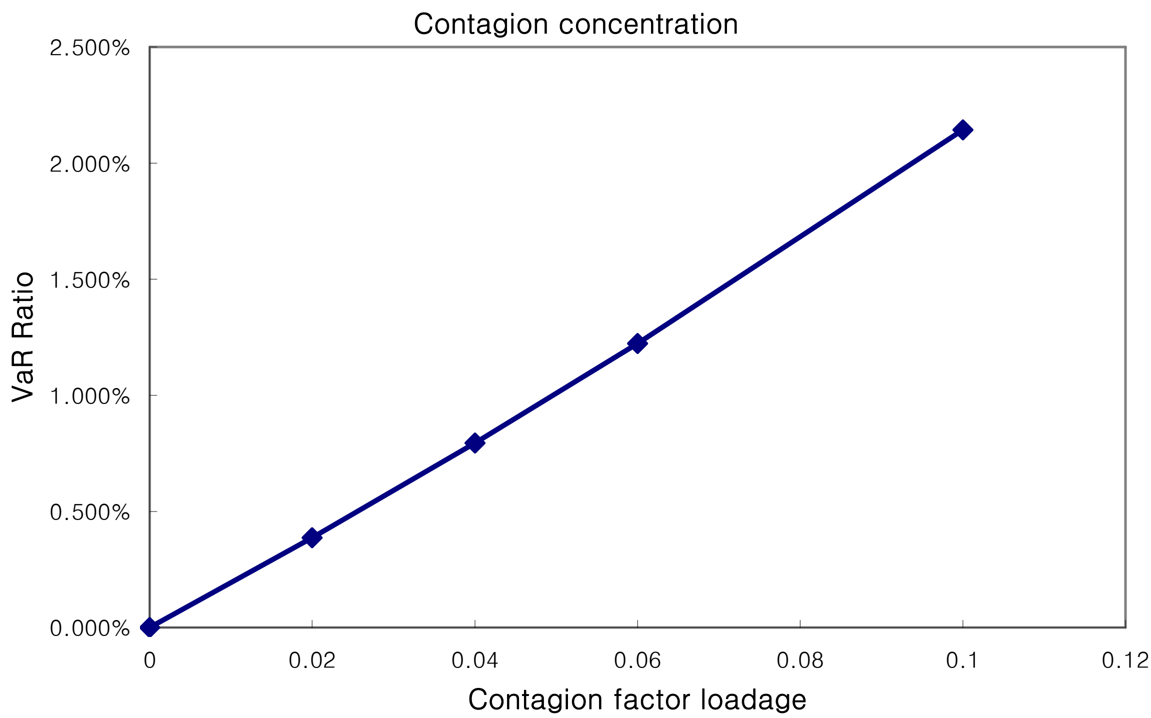
< Figure 2 > Relationship between Herfindahl index and exposure concentration

Portfolio 5~10 have the same PD, LGD, EAD, maturity, and systematic factor Loading. These portfolio have the same Herfindahl index (=0.001455%) but different industry correlation. <Figure 3> shows relationship between industry correlation coefficient and sector concentration risk on portfolio 5~10 (have the same Herfindahl index). As I pointed out, Herfindahl index and exposure concentration has linear relationship whereas Herfindahl index does not have linear relationship with industry correlation. This means that industry correlation does not have relationship with Herfindahl index. Therefore, this can be concluded as Herfindahl index from portfolio does not explain portfolio's sector concentration.



< Figure 3 > Relationship between correlation coefficient among industries and sector concentration risk

What about contagion factor? Portfolio 11~15 are related to contagion factor. Portfolio 5~10 have the same PD, LGD, EAD, maturity, systematic factor Loading, Correlation coefficient among industries but have different contagion factor Loading. These portfolios may have different contagion factor for each obligors, however, I applied the same contagion factor for them. <Figure 4> shows how contagion concentration risk changes along with contagion factor Loading changes. Contagion factor Loading has linear relationship with contagion concentration risk. However, portfolio 11~15 have the same Herfindahl index, which indicates that Herfindahl index cannot explain contagion concentration risk.



< Figure 4 > Relationship between contagion factor Loading and contagion concentration risk

, Herfindahl index, which is mostly used for concentration indication, is proper to explain exposure concentration. However, there is limitation that it cannot explain common or correlated risk factor. Therefore, to measure or recognize concentration risk from common risk factors, model, which reflects common risk factors, will be requested and EMFA model is the one.

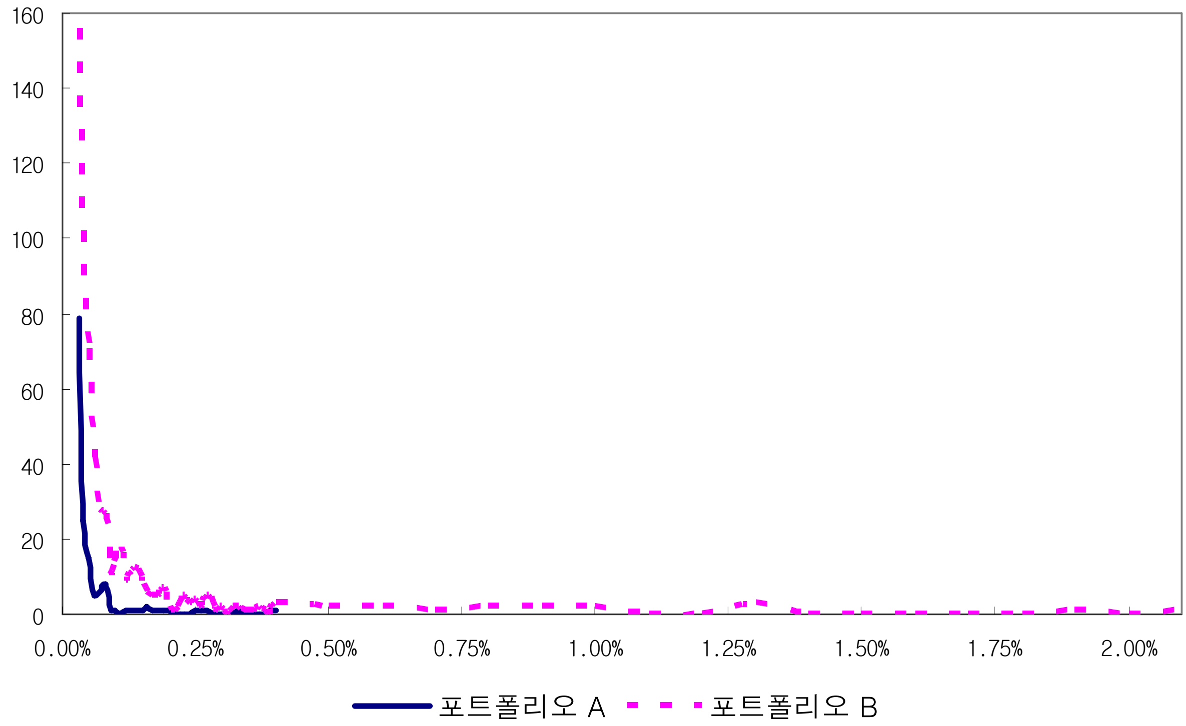
5.2. Analysis Case B

5.2.1. Data

Make two portfolios (A and B) which have the same inputs but different EAD distributions. The difference between them, as it is shown at <Table 4> and <Figure 2>, portfolio B had wider EAD weight than portfolio A. In other words, portfolio B is consisted of obligors, which have relatively small or big exposure. Therefore it is more concentrated than portfolio A.

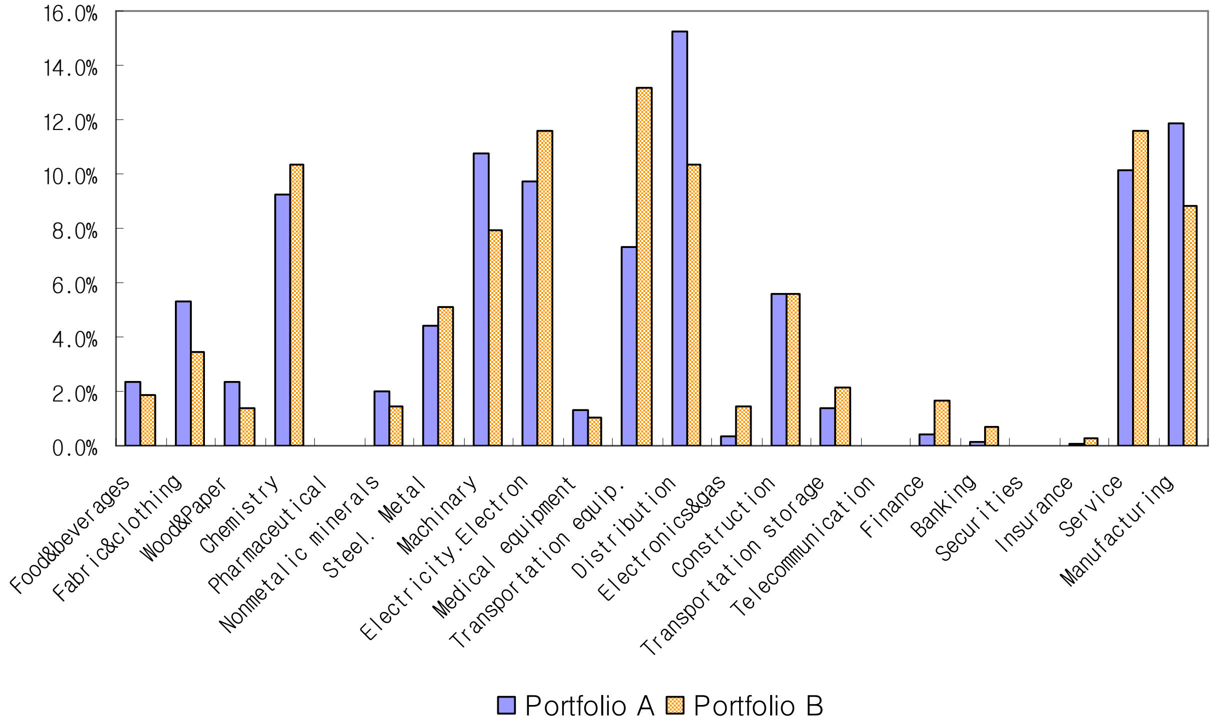
Classification	Portfolio A	Portfolio B
Average	0.002000%	0.002000%
Standard deviation	0.005310%	0.023018%
Maximum EAD weight	0.4040%	2.1074%
Minimum EAD weight	0.00024%	0.00000023%

< Table 6 > EAD weight statistics for portfolio A and B



< Figure 5 > EAD weight distribution of portfolio A and B (exclude under 0.025%)

Following shows EAD weight by industry for portfolio A and B. Both portfolios have high percentages in chemistry, machinery, electricity&electron, transfortation equipment, distribution, service, manufacturing field.



< Figure 6 > EAD distribution by industry for portfolio A and B

Classification	Chemistry		Machinery		Electricity & electron	
	A	B	A	B	A	B
Average	0.0025%	0.0028%	0.0019%	0.0014%	0.0023%	0.0028%
Standard deviation	0.0060%	0.0248%	0.0029%	0.0095%	0.0076%	0.0372%
Max. EAD weight	0.2723%	1.2557%	0.0752%	0.3790%	0.4040%	2.1074%
Min. EAD weight	0.0002%	0.0000%	0.0002%	0.0000%	0.0002%	0.0000%
Herfindahl index	0.1805%	2.1507%	0.0577%	0.8205%	0.2777%	4.3524%

Classification	Trans. Equip.		Distribution		Service		Manufacturing	
	A	B	A	B	A	B	A	B
Average	0.0035%	0.0063%	0.0014%	0.0010%	0.0019%	0.0022%	0.0020%	0.0015%
Standard	0.0112%	0.0521%	0.0036%	0.0155%	0.0074%	0.0305%	0.0028%	0.0079%

deviation								
Max. EAD weight	0.3867%	1.9224%	0.2510%	1.2646%	0.3337%	1.3383%	0.0472%	0.2762%
Min. EAD weight	0.0002%	0.0000%	0.0002%	0.0000%	0.0002%	0.0000%	0.0002%	0.0000%
Herfindahl index	0.5330%	3.3271%	0.0679%	2.3868%	0.3014%	3.7346%	0.0499%	0.4873%

< Table 7 >Statistics for EAD weight

5.2.2. Analysis result

Following <Table 8> and <Table 9> show exposure, sector and contagion concentration and VaR ratio and expected shortfall ratio of total credit concentration of portfolio A and B.

Classification	Herfindahl index	Exposure (VaR Ratio)	Sector (VaR Ratio)	Contagion (VaR Ratio)	Total credit (VaR Ratio)
Portfolio A	0.000160967	0.050618%	0.151441%	0.000316%	0.202375%
Portfolio B	0.002669228	0.690824%	1.109445%	0.004715%	1.804984%
Difference(B-A)	0.002508261	0.640206%	0.958003%	0.004399%	1.602609%

< Table 8 > VaR ratio at exposure, sector, contagion and total credit concentration of portfolio A and B

Classification	Herfindahl index	Exposure (ES Ratio)	Sector (ES Ratio)	Contagion (ES Ratio)	Total credit (ES Ratio)
Portfolio A	0.000160967	0.060397%	0.191392%	0.000416%	0.252206%
Portfolio B	0.002669228	0.802022%	1.454739%	0.005980%	2.262741%
Difference(B-A)	0.002508261	0.741625%	1.263347%	0.005564%	2.010535%

< Table 9 > Expected shortfall ratio at exposure, sector, contagion and total credit concentration of portfolio A and B

Result that we compared portfolio A and B which have different EAD distribution but the same inputs, EAD distribution makes big difference in concentration risk. (portfolio B has larger risk than A). This means that exposure distribution affects significantly on concentration risk. That is, credit concentration risk can be managed by exposrues's weight.

In <Table 10>, when you look at credit concentration risk which has huge exposure, machinery and manufacturing has similar Herfindahl index. However, machinery has bigger concentration risk. When we compare EAD weight statistics of machinery and manufacturing, we can see the reason. It is because machinery obligors, who have large EAD weight, appear larger than manufacturing's. That is, we can say in credit concentration risk management by industry, exposure weight is a key management point.

Classification	**Chemistry**	**Machinery**	**Electricity&electron**
Exposure	0.0445%	0.0036%	0.0986%
Sector	0.2433%	0.0242%	0.2599%
Contagion	5.4675%	0.0068%	12.8257%
Total concentration	13.1933%	1.4733%	17.2652%
Herfindahl index	0.1805%	0.0577%	0.2777%

Classification	**Transportation equipment**	**Distribution**	**Service**	**Manufacturing**
Exposure	0.1413%	0.0381%	0.1083%	0.0010%
Sector	0.4465%	0.1379%	0.1018%	0.0005%
Contagion	58.5412%	1.9923%	5.8896%	0.0003%
Total concentration	21.4126%	13.2294%	10.5434%	0.0844%
Herfindahl index	0.5330%	0.0679%	0.3014%	0.0499%

< Table 10 > Portfolio A's concentration risk ratio for 7 industries (VaR Ratio)

Especially, chemistry and distribution industry show this kind of characteristics. While distribution industry and chemistry's total concentration show similarity, in Herfindahl index, chemistry has three times bigger than distribution industry. When we see the structure of chemistry and distribution industry's credit concentration risk, sector and contagion concentration seem to be a reason for this. Especially, chemistry has obligors who are under the same industry sector and also has obligors who have huge exposures. Therefore, chemistry industry's management key point is decreasing same sector obligors and huge exposure obligors. On the contrary, distribution industry has evenly distributed exposres for its sector. Rather prevent industry's exposure expanding than obligor level management for distribution industry. Consequently, as we recognize credit concentration risk by exposure, sector and contagion concentration, which are subdivided format, there is a good point that we also can recognize managerial key point by industry through risk measurement.

5.2.3. The result of adjustment of asset correlation

Calculate adjusted coefficient (see 4.3) for portfolio A and B at 5.2.2. Following table is the result. Portfolio A's exposure distribution is not concentrated as portfolio B. Therefore portfolio has low asset correlation.

Classification	Portfolio correlation coefficient		Adjusted coefficient
	IRB	EMFA	
Portfolio A	0.13167830	0.06158885	0.07008945
Portfolio B	0.15835989	0.12205329	0.0363066

< Table 11 > Asset correlation adjusted coefficient

Classification		Exposure concentration (VaR Ratio)	Sector concentration (VaR Ratio)	Contagion concentration (VaR Ratio)	Total credit concentration (VaR Ratio)
Before adjustment	Portfolio A	0.050618%	0.151441%	0.000316%	0.202375%
	Portfolio B	0.690824%	1.109445%	0.004715%	1.804984%
After adjustment	Portfolio A	0.050618%	0.637973%	0.000362%	0.688952%
	Portfolio B	0.690824%	1.601148%	0.005055%	2.297027%

< Table 12 > Asset correlation adjusted coefficient applying result

As we see at the <Table 11>, when we adjust asset correlation exposure concentration does not show much difference while sector concentration does.

Along with numbers of systematic factors, correlation among them, and their loadage, credit concentration risk is changed. There is a possibility to measure credit concentration risk too small as reducing those factors, correlation and loadage. Especially, the number of systematic factors (industries) can be different due to the purpose of industry management by individual financial institutions. For that reason, as classification standard among individual financial institution is different, there is limitation that credit concentration risk's comparability cannot meet FSS's expectation.

Applying asset correlation adjustment coefficient is the right method for the purpose, which increases comparability. However, as we can see from the analysis result, when number of systematic factors, their correlations and loadage are the same and exposure distribution is different, the portfolio which has even distribution must have small asset correlation but apply adjusted coefficient, there is a problem that credit concentration risk measured relatively unfavorable.

Conclusion, asset correlation adjustment coefficient contributes to increase comparability of credit concentration among financial institutions while having limitation, which offsets individual portfolio's degree concentration Therefore, a local standard for systematic factors should be developed to recognize individual portfolio's characteristics and increase comparability among financial institutions.

6. Conclusion

By this time credir concentration risk measurement model was developed in each field which is exposure concentration, sector concentration and contagion concentration. However, there was no such a model which integrates and measures all these three concentrations. I present a model which integrates exposure, sector and contagion concentration. Emmer & Tasche(2005) Pykhtin(2004), Egloff and others (2004), were studied for each exposure concentration, sector concentration and contagion concentration and EMFA model integrates all these people's work.

Exposure concentration has been shown linear relationship with Herfindahl index. Here we have a point that using Herfindahl index, we could measure exposure concentration and manage it. This result is also accord with Gordy (2003)'s result. However, even they have the same Herfindahl index, due to industrial correlation and contagion factor, credit concentration risk has different aspect. This means that Herfindahl index cannot explain credit concentration risk by common or correlated risk factor. Industrial correlation and credit concentration risk have non-linear relationship and with contagion factor credit concentration risk have linear relationship. It means that credit concentration risk by common or correlated risk factor has complicated structure. However, exposure wieht gives more effect than common or correlated risk factor[5].

EMFA model has closed form solution which is easy to implement and it is an extended form of IRB model which has consistency with Basel II frame work. In addition to that it could recognize and measure various credit concentration risk.

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[5] PD, LGD are one of the most important factors which affects on credit concentration risk. However, PD, LGD are the common important factors in credit risk therefore we've mentioned exposure weight as a main factor other than these.

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Appendix

7.1. A : Extended Multi-Factor Adjustment model induce

EMFA is different from Pykhtin (2004)'s exposure concentration (name concentration). To explain this, we start from Pykhtin (2004)'s expression (2.2.1). $l'(y), l''(y)$ were explained already. Expression (2.2.1) did not explain about $v(y)$ which is actually $\text{var}(L|\overline{Y}=y)=v(y)$. $v(y)$ means variance and $y=N^{-1}(1-\beta)$ therefore, it is expressed as $\text{var}(L|\overline{Y}=y)=v(y)$ and $v(y)$ is following;

$$\text{var}\left(L\middle|\overline{Y}=N^{-1}(1-\beta)\right)=\sum_{i=1}^{N}\text{var}\left(L_i\middle|\overline{Y}=N^{-1}(1-\beta)\right)+\sum_{\substack{i=1\\ i\neq j}}^{N}\sum_{j=1}^{N}Cov\left[L_i,L_j\middle|\overline{Y}=N^{-1}(1-\beta)\right] \quad (A.1)$$

L , which is not using infinitely-fine grained assumption is expressed as following;

$$L=\sum_{i=1}^{N}L_i=\sum_{i=1}^{N}e_i\cdot LGD_i\cdot 1_{\left[r_i\leq N^{-1}(PD_i)\right]} \quad (A.2)$$

$1_{\left[r_i\leq N^{-1}(PD_i)\right]}$ means indication function when there is default, "1" no default "0."

Expression (A.2)'s $1_{\left[r_i\leq N^{-1}(PD_i)\right]}$ becomes when asset i has default then, it will be 1 and other wise it will be 0. Therefore the probability of $r_i\leq N^{-1}(PD_i)$ is conditional probaibility of default of asset i. Probability of default($PD_{i`}$) is given and in the expression $y=N^{-1}(1-\beta)$, expression (A.1)'s the first paragraph $\left(\sum_{i=1}^{N}\text{var}\left(L_i\middle|\overline{Y}=N^{-1}(1-\beta)\right)\right)$ became binomial distribution's variance. Therefore,, $\sum_{i=1}^{N}\text{var}\left(L_i\middle|\overline{Y}=N^{-1}(1-\beta)\right)$ is the same as following, the result is the same as Emmer & Tasche(2005)'s but the LGD is randon variable.

$$\sum_{i=1}^{N}\mathrm{var}\left(L_i\middle|\overline{Y}=N^{-1}(1-\beta)\right)$$
$$=\sum_{i=1}^{N}e_i^2\cdot\left\{LGD_i^2\cdot\left[N\left(\frac{N^{-1}(PD_i)-a_iy}{\sqrt{1-a_i^2}}\right)-N\left(\frac{N^{-1}(PD_i)-a_iy}{\sqrt{1-a_i^2}}\right)N\left(\frac{N^{-1}(PD_i)-a_iy}{\sqrt{1-a_i^2}}\right)\right]+\sigma_i^2\,\hat{p}_i(y)\right\}$$

(A.3)

In IRB model, each risk for asset i and asset j in asset profitability aspect expressed as following; $p\left(\varsigma_i<\frac{1}{\sqrt{1-a_i^2}}N^{-1}(PD_i)-\sqrt{\frac{a_i^2}{1-a_i^2}}\right)$, $p\left(\varsigma_j<\frac{1}{\sqrt{1-a_j^2}}N^{-1}(PD_j)-\sqrt{\frac{a_j^2}{1-a_j^2}}\right)$.

Therefore, convariance for various loss of asset can be expressed as following;

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\i\neq j}}^{N}Cov\left[L_i,L_j\middle|\overline{Y}=N^{-1}(1-\beta)\right]$$
$$=\sum_{i=1}^{N}\sum_{\substack{j=1\\i\neq j}}^{N}\left\{E\left[L_iL_j\middle|\overline{Y}=N^{-1}(1-\beta)\right]-E\left[L_i\middle|\overline{Y}=N^{-1}(1-\beta)\right]E\left[L_j\middle|\overline{Y}=N^{-1}(1-\beta)\right]\right\}$$
$$=\sum_{i=1}^{N}\sum_{j=1}^{N}e_ie_jLGD_iLGD_j\left\{N_2\left(\frac{N^{-1}(PD_i)-a_iy}{\sqrt{1-a_i^2}},\frac{N^{-1}(PD_j)-a_jy}{\sqrt{1-a_j^2}},\rho_{ij}^{Y+C}\right)-N\left(\frac{N^{-1}(PD_i)-a_iy}{\sqrt{1-a_i^2}}\right)N\left(\frac{N^{-1}(PD_j)-a_jy}{\sqrt{1-a_j^2}}\right)\right\}$$
$$-\sum_{i=1}^{N}e_i^2\cdot LGD_i^2\left\{N_2\left(\frac{N^{-1}(PD_i)-a_iy}{\sqrt{1-a_i^2}},\frac{N^{-1}(PD_i)-a_iy}{\sqrt{1-a_i^2}},\rho_{ii}^{Y+C}\right)-N\left(\frac{N^{-1}(PD_i)-a_iy}{\sqrt{1-a_i^2}}\right)^2\right\}$$

(A.4)

Expression (A.3) is v_{GA} and expression (A.4) is $v_\infty(y)$.

$v_{GA}(y)$ related to exposure concentration and $v'_{GA}(y)$, related to sector concentration are $v_\infty(y)$ and $v'_\infty(y)$, which are expressed as following;

$$v_{GA}=\sum_{i=1}^{N}e_i^2\cdot\left\{LGD_i^2\cdot\left[p_i(y)-p_i(y)p_i(y)\right]+\sigma_i^2\,p_i(y)\right\} \quad \text{(A.5)}$$

$$v'_{GA}(y)=\sum_{i=1}^{N}e_i^2\cdot p'_i(y)\cdot\left(LGD_i^2\cdot\left[1-2\cdot p_i(y)\right]+\sigma_i^2\right) \quad \text{(A.6)}$$

$$v_{\infty}(y)=$$
$$=\sum_{i=1}^{N}\sum_{j=1}^{N}e_i e_j LGD_i LGD_j\left\{N_2\left(\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}},\frac{N^{-1}(PD_j)-a_j y}{\sqrt{1-a_j^2}},\rho_{ij}^{Y+C}\right)-N\left(\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}}\right)N\left(\frac{N^{-1}(PD_j)-a_j y}{\sqrt{1-a_j^2}}\right)\right\}$$
$$-\sum_{i=1}^{N}e_i^2\cdot LGD_i^2\left\{N_2\left(\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}},\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}},\rho_{ii}^{Y+C}\right)-N\left(\frac{N^{-1}(PD_i)-a_i y}{\sqrt{1-a_i^2}}\right)^2\right\} \quad \text{(A.7)}$$

$$v'_{\infty}(y)=2\sum_{i=1}^{N}\sum_{j=1}^{N}e_i\cdot e_j\cdot LGD_i\cdot LGD_j\cdot p'(y)\left\{N\left(\frac{N^{-1}[p_j(y)]-\rho_{ij}^{Y+C}N^{-1}[p_i(y)]}{\sqrt{1-\left(\rho_{ij}^{Y+C}\right)^2}}\right)-p_j(y)\right\}$$
$$-2\sum_{i=1}^{N}e_i^2\cdot LGD_i^2\cdot p'(y)\left\{N\left(\frac{N^{-1}(PD_i)-\rho_{ii}^{Y+C}y}{\sqrt{1-\left(\rho_{ii}^{Y+C}\right)^2}}\right)-p_i(y)\right\} \quad \text{(A.8)}$$

By now, I explained about a model, which measures total credit concentration risk for whole portfolio. To measure individual asset unit's credit concentration risk, using marginal VaR. When I differentiate expression (2.2.1) by exposure weight, the result is the same as following;

$$\frac{d(l_\beta(L))}{de_i}-\frac{d(l_\beta(\bar{L}))}{de_i}=\frac{d(l_\beta(L))}{de_i}-LGD_i\cdot p_i(y)$$
$$\approx -\frac{1}{2}\left[\frac{B\cdot l'-v'\cdot A}{(l')^2}-\frac{C\cdot l'-l''\cdot A}{(l')^2}\cdot\frac{v}{l'}-\left(\frac{l''}{l'}+y\right)\left(\frac{D\cdot l'-v\cdot A}{(l')^2}\right)\right] \quad \text{(A.9)}$$

$$A=\frac{dl'(y)}{de_i}=LGD_i\cdot p_i'(y)$$
$$B=\frac{dv'(y)}{de_i}=\frac{dv'_{GA}(y)}{de_i}+\frac{dv'_{\infty}(y)}{de_i}$$
$$C=\frac{dl''(y)}{de_i}=LGD_i\cdot p_i''(y)$$
$$D=\frac{dv(y)}{de_i}=\frac{dv_{GA}(y)}{de_i}+\frac{dv_{\infty}(y)}{de_i}$$

In expression (A.9), B and D are calculated as we differentiate expression (2.2.1), (2.2.2), (2.2.3), (2.2.4) by exposure weight.

$$\frac{dv_{GA}(y)}{de_i} = 2\cdot e_i \cdot \left(LGD_i^2 \cdot [p_i(y) - p_i(y)p_i(y)] + \sigma_i^2 p_i(y)\right) \quad \text{(A.10)}$$

$$\frac{dv'_{GA}(y)}{de_i} = 2\cdot e_i \cdot p'_i(y)\cdot \left(LGD_i^2 \cdot [1 - 2\cdot p_i(y)] + \sigma_i^2\right) \quad \text{(A.11)}$$

$$\begin{aligned}\frac{dv_\infty(y)}{de_i} &= 2\cdot LGD_i \cdot \sum_{j=1}^{N} e_j LGD_j \left\{N_2\left(N^{-1}[p_i(y)], N^{-1}[p_j(y)], \rho_{ij}^{Y+C}\right) - p_i(y)p_j(y)\right\} \\ &\quad - 2\cdot e_i \cdot LGD_i^2 \left\{N_2\left(N^{-1}[p_i(y)], N^{-1}[p_i(y)], \rho_{ii}^{Y+C}\right) - p_i(y)^2\right\}\end{aligned} \quad \text{(A.12)}$$

$$\begin{aligned}\frac{dv'_\infty(y)}{de_i} &= 2\cdot 2\cdot LGD_i \cdot \sum_{j=1}^{N} e_j \cdot LGD_j \cdot p'_i(y)\left\{N\left(\frac{N^{-1}[p_j(y)] - \rho_{ij}^{Y+C} N^{-1}[p_i(y)]}{\sqrt{1-(\rho_{ij}^{Y+G})^2}}\right) - p_j(y)\right\} \\ &\quad - 2\cdot e_i \cdot p'_i(y)\left(LGD_i^2\left[2N\left(\frac{N^{-1}[p_j(y)] - \rho_{ii}^{Y+C} N^{-1}[p_i(y)]}{\sqrt{1-(\rho_{ii}^{Y+C})^2}}\right) - 2p_i(y)\right]\right)\end{aligned} \quad \text{(A.13)}$$

$$\left(\because N^{-1}(p_i(y)) = \frac{N^{-1}(PD_i) - a_i y}{\sqrt{1-a_i^2}}.\right)$$

Expression (A.10) and (A.11) are related to exposure concentration and expression (A.12) and (A.13) are related to sector concentration.

When I express (A.9) as simple,

$$\frac{d(l_\beta(L))}{de_i} - \frac{d(l_\beta(\overline{L}))}{de_i} = F\left(PD, LGD, e, \rho^{Y+C}\right) \quad \text{(A.14)}$$

Contagion concentration can be calculated as deduct multi factor risk amount from mutifactor and contagion concentration risk amount.

$$Contagionconcentration = F\left(PD, LGD, e, \rho^{Y+C}\right) - F\left(PD, LGD, e, \rho^{Y}\right) \quad \text{(A.15)}$$

ρ^{Y} : Conditional asset correlation reflecting multi-factor

ρ^{Y+C} : Conditional asset correlation reflecting multi-factor and contagion factor

Lastly, result from sector concentration expression, when it is plus, it is concentration risk and when it is minus, it is variance effect. This is how to get sector concentration risk. In addition to that, this will be calculated by individual asset unit, which is easy to reflect maturity effect from Basel II.

As Pykhtin (2004) suggested, expected shortfall can be calculated using EMFA. When we differentiate expressions (2.2.7) and (2.2.8) from chapter 2.2, they are the same as following. These expressions are expressed by (A.9), (A.10), and (A.12). Therefore, we can calculate expected shortfall from EMFA.

$$\frac{dES_\beta(\overline{L})}{de_i} = \frac{1}{1-\beta} \cdot LGD_i \cdot N_2\left[N^{-1}(PD_i), N^{-1}(1-\beta), a_i\right] \qquad \text{(A.16)[6]}$$

$$\frac{d(\Delta ES_\beta(L))}{de_i} = -\frac{1}{2(1-\beta)} n(y)\left(\frac{D \cdot l' - v \cdot A}{(l')^2}\right)\Bigg|_{y = N^{-1}(1-\beta)} \qquad \text{(A.17)}$$

$$A = \frac{dl'(y)}{de_i} = LGD_i \cdot p_i'(y)$$

$$D = \frac{dv(y)}{de_i} = \frac{dv_{GA}(y)}{de_i} + \frac{dv_\infty(y)}{de_i}$$

$$\frac{dv_{GA}(y)}{de_i} = 2 \cdot e_i \cdot \left(LGD_i^2 \cdot [p_i(y) - p_i(y)p_i(y)] + \sigma_i^2 p_i(y)\right)$$

$$\frac{dv_\infty(y)}{de_i} = 2 \cdot LGD_i \cdot \sum_{j=1}^{N} e_j LGD_j \left\{N_2\left(N^{-1}[p_i(y)], N^{-1}[p_j(y)], \rho_{ij}^{Y+C}\right) - p_i(y)p_j(y)\right\}$$
$$- 2 \cdot e_i \cdot \sum_{i=1}^{N} LGD_i^2 \left\{N_2\left(N^{-1}[p_i(y)], N^{-1}[p_i(y)], \rho_{ii}^{Y+C}\right) - p_i(y)^2\right\}$$

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[6] When it follows Basel II's IRB expression, expression (A.18) is the same as following;.

$$\frac{dES_\beta(\overline{L})}{de_i} = LGD_i \cdot \left[\frac{1}{1-\beta} \cdot N_2\left[N^{-1}(PD_i), N^{-1}(1-\beta), a_i\right] - PD_i\right] \times MA_i,$$ MA_i is maturity effect.

7.2. B : Correlations among industries

	F&B	Fabric Cloth ing	Paper wood	Chem	Medi cine	Non Meta Lic Mine ral	Steel metal	Mach-inery	Elec tri city	Medi Cal equip	Trans porta tion	Dist rebu tion	Elec Tri city &gas	Cons truc tion	Trans Porta Tion& Stora -ge	Comm Unic ation	Fina ncial Insti tution	Bank	Sec.	Insur avnce	Ser vice	Manu Factu re
F & B	1.00	0.57	0.66	0.77	0.55	0.60	0.52	0.69	0.59	0.56	0.58	0.63	0.54	0.64	0.65	0.45	0.66	0.64	0.57	0.63	0.63	0.72
Fab ric Clot ing	0.57	1.00	0.68	0.59	0.53	0.47	0.50	0.67	0.52	0.56	0.57	0.60	0.36	0.53	0.62	0.22	0.56	0.49	0.56	0.54	0.47	0.62
Paper Wood	0.66	0.68	1.00	0.78	0.55	0.67	0.63	0.75	0.62	0.62	0.65	0.72	0.49	0.74	0.70	0.33	0.68	0.61	0.66	0.63	0.57	0.74
Chem	0.77	0.59	0.78	1.00	0.55	0.74	0.74	0.78	0.69	0.65	0.72	0.77	0.50	0.71	0.82	0.48	0.73	0.67	0.66	0.72	0.66	0.84
Medi cine	0.55	0.53	0.55	0.55	1.00	0.50	0.38	0.66	0.36	0.53	0.35	0.40	0.24	0.50	0.42	0.35	0.42	0.26	0.52	0.49	0.66	0.49
Non Meta Lic Mine ral	0.60	0.47	0.67	0.74	0.50	1.00	0.54	0.70	0.45	0.50	0.68	0.62	0.41	0.62	0.59	0.24	0.62	0.58	0.58	0.67	0.49	0.61
Steel Metal	0.52	0.50	0.63	0.74	0.38	0.54	1.00	0.58	0.57	0.55	0.59	0.61	0.40	0.56	0.65	0.50	0.65	0.59	0.63	0.58	0.50	0.71
Machi nery	0.69	0.67	0.75	0.78	0.66	0.70	0.58	1.00	0.61	0.68	0.76	0.68	0.40	0.70	0.71	0.35	0.68	0.59	0.67	0.69	0.66	0.76
Elect Ricit y	0.59	0.52	0.62	0.69	0.36	0.45	0.57	0.61	1.00	0.67	0.69	0.67	0.40	0.54	0.65	0.52	0.65	0.55	0.68	0.68	0.53	0.96

	F&B	Fabric Clothing	Paper wood	Chem	Medicine	Non Metalic Mineral	Steel metal	Mach-inery	Electricity	Medical equip	Transportation	Distrebution	Electricity &gas	Construction	Trans Portation& Storage	Communication	Financial Institution	Bank	Sec.	Insuravnce	Service	Manufacture
Medical equip	0.56	0.56	0.62	0.65	0.53	0.50	0.55	0.68	0.67	1.00	0.56	0.59	0.29	0.59	0.61	0.43	0.60	0.46	0.66	0.61	0.65	0.73
TransPortation	0.58	0.57	0.65	0.72	0.35	0.68	0.59	0.76	0.69	0.56	1.00	0.72	0.40	0.63	0.70	0.38	0.74	0.69	0.71	0.74	0.47	0.81
Distribution	0.63	0.60	0.72	0.77	0.40	0.62	0.61	0.68	0.67	0.59	0.72	1.00	0.44	0.66	0.72	0.37	0.71	0.65	0.66	0.66	0.56	0.77
Electricity& Gas	0.54	0.36	0.49	0.50	0.24	0.41	0.40	0.40	0.40	0.29	0.40	0.44	1.00	0.59	0.46	0.42	0.53	0.50	0.44	0.46	0.29	0.47
ConstRuction	0.64	0.53	0.74	0.71	0.50	0.62	0.56	0.70	0.54	0.59	0.63	0.66	0.59	1.00	0.62	0.44	0.74	0.64	0.71	0.68	0.57	0.67
Transportation& Storage	0.65	0.62	0.70	0.82	0.42	0.59	0.65	0.71	0.65	0.61	0.70	0.72	0.46	0.62	1.00	0.40	0.69	0.61	0.64	0.68	0.55	0.77
CommuNication	0.45	0.22	0.33	0.48	0.35	0.24	0.50	0.35	0.52	0.43	0.38	0.37	0.42	0.44	0.40	1.00	0.56	0.48	0.54	0.54	0.44	0.55
FinanCial Institution	0.66	0.56	0.68	0.73	0.42	0.62	0.65	0.68	0.65	0.60	0.74	0.71	0.53	0.74	0.69	0.56	1.00	0.94	0.88	0.87	0.51	0.76
Bank	0.64	0.49	0.61	0.67	0.26	0.58	0.59	0.59	0.55	0.46	0.69	0.65	0.50	0.64	0.61	0.48	0.94	1.00	0.72	0.76	0.38	0.66

	F&B	Fabric Clothing	Paper wood	Chem	Medicine	Non MetaLic Mineral	Steel metal	Mach-inery	Electricity	Medi Cal equip	Transportation	Distrebution	Elec Tricity &gas	Construction	Trans PortaTion& Stora-ge	CommUnication	Financial Institution	Bank	Sec.	Insuravnce	Service	ManuFacture
Sec.	0.57	0.56	0.66	0.66	0.52	0.58	0.63	0.67	0.68	0.66	0.71	0.66	0.44	0.71	0.64	0.54	0.88	0.72	1.00	0.84	0.56	0.76
Insurance	0.63	0.54	0.63	0.72	0.49	0.67	0.58	0.69	0.68	0.61	0.74	0.66	0.46	0.68	0.68	0.54	0.87	0.76	0.84	1.00	0.53	0.77
Service	0.63	0.47	0.57	0.66	0.66	0.49	0.50	0.66	0.53	0.65	0.47	0.56	0.29	0.57	0.55	0.44	0.51	0.38	0.56	0.53	1.00	0.63
ManufActure	0.72	0.62	0.74	0.84	0.49	0.61	0.71	0.76	0.96	0.73	0.81	0.77	0.47	0.67	0.77	0.55	0.76	0.66	0.76	0.77	0.63	1.00